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ON A POSSIBLE STRUCTURE OF THE SURFACE LAYER OF THE MOON

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SUMMARY

An analytical expression has been derived for an effective thermal conduction of bodies of various structures. The solution of an inverse problem is considered, i. e. the determination of the structure of a body by an effective coefficient of thermal conduction.

Consideration is also given to a possible structure of the lunar surface layer, based on the analysis of radioastronomical data relative to the parameter $\gamma = (\lambda\rho c)^{-1/2}$ and some additional assumptions.

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The study of generalized conduction (thermal conduction, electrical conductivity and others) of diphase systems was performed by numerous investigators. Analytical dependences are proposed, which link the physical and structural parameters with the effective transfer coefficients for separate forms of diphase systems' structures [1 - 4], such as the granular structure of a solid with closed and communicating pores.

The solution of the inverse problem is of interest; namely, the determination by the effective transfer coefficient (for example, the effective heat conductivity of a diphase system) of its structure. Such a problem has arisen, in particular, when studying the structure of the surface layer of the Moon. According to precision measurements of temperature during lunations, the following value of the parameter γ was obtained: [5]:

$$\gamma = (\lambda\rho c)^{-\frac{1}{2}} \simeq 400$$

Assuming the density of lunar matter from the surface layer $\rho > 0.4$ g cm^{-3}, which corresponds to the density of terrestrial volcanic rocks (such as tuff)

* O VOZMOZHNOY STRUKTURE POVERKHNOSTNOGO SLOYA LUNY

and the heat capacity of the Moon's matter $c \sim 0.2\ cal \cdot g^{-1}\ deg^{-1} = 840$ joule$\cdot kg^{-1}\ deg^{-1}$, we find by the value of γ the effective heat conductivity of the matter of Moon's superficial layer:

$$\lambda \simeq 8 \cdot 10^{-5}\ cal \cdot sec^{-1} cm^{-1} \cdot deg^{-1} = 0.035\ w \cdot m^{-1} \cdot deg^{-1}.$$

Let us compare the obtained value of the effective heat conductivity with the heat conductivity of various structures in conditions of deep vacuum. The heat conductivity factor for a mineral dust in vacuum and normal temperature oscillates within the limits from 0.003 to 0.1 $w \cdot m^{-1}\ deg^{-1}$ [8, 9]; that of a highly porous solid in vacuum for a density $\rho \approx 0.4 \div 0.6\ g \cdot cm^{-3}$ varies within the limits from 0.07 to 0.7 $w \cdot m^{-1}\ deg^{-1}$. Consequently, for the lunar surface layer it should occupy an intermediate position between the heat conductivity factors of mineral dust and of a solid porous body of mineral origin in conditions of deep vacuum. These results allow us to assume the existence of intermediate structures, which in the following we shall designate as "dendritic". A sketch of such a structure is given in the Fig.1 , where the dendritic structure is represented in the form of a solid body with communicating pores (Fig.1a) of which the skeleton consists of beams with variable cross section.

The dendritic structures may be obtained artificially. Description of experiments is brought out in the work [6], in which "caking" of separate particles of granular systems into conglomerates is observed in conditions of deep vacuum and at temperatures of the order of 500°K. At the same time the surface of the contact along which "caking" of particles takes place is $10^2 \div 10^4$ times smaller than the area of their maximum cross section.

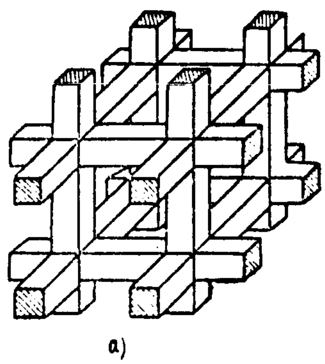
Let us find the effective heat conductivity of a dendritic structure in conditions of deep vacuum (the gas-filler pressure in pores is $< 1 \cdot 10^{4}$ mm Hg) at temperatures from 0° to 300°K. In this case the radial and molecular heat transfer is negligibly small by comparison with the heat transfer through the hard skeleton. To facilitate the calculations we shall take advantage of the notion of "elementary mesh" (of minimum volume, maintaining all the properties of dendritic structure), by multiple repetition of which it is possible to obtain a system with dendritic structure [3].

To begin with we shall consider the effective heat conductivity of a solid with communicating pores, of which the skeleton consists of beams with constant cross section (Fig.1a). The shape of the elementary mesh is shown in Fig.2a. The effective heat conductivity of such structures in conditions of deep vacuum may be determined by the formula [3]

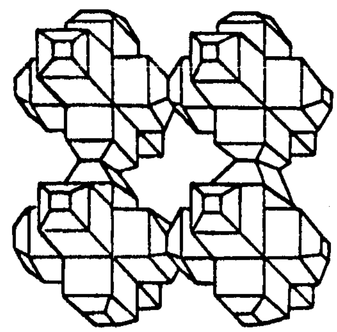
$$\lambda_{eff} = \lambda_1 x^2, \quad x = \Delta/L, \qquad (1)$$

where λ_1 is the heat conductivity of skeleton's material, Δ is the dimension of of beam's cross section, L is the dimension of the elementary mesh (see Fig.1b). Parameter $\underline{x}$ depends unambiguously on the porousness of the material; this dependence is brought up below (see (8)).

In real diphase systems pores are chaotically distributed by volume. Such a system may be schematically represented in the form of cubic skeletons shifted relative to one another (Fig.1c). An elementary mesh of such a structure is represented in Fig.2b. The thermal flux enters the elementary mesh through the total cross section of a beam with the area $S_{mx} = \Lambda^2$. The shift of pores increases the path length of the thermal flux along the conducting phase, i. e. along the solid skeleton, increasing by the same token the thermal resistance of the elementary mesh.



a)



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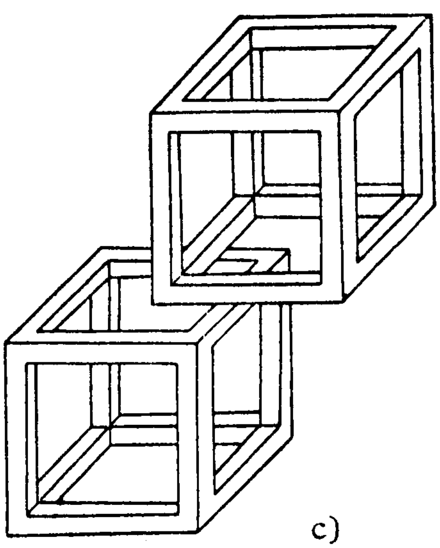


c)

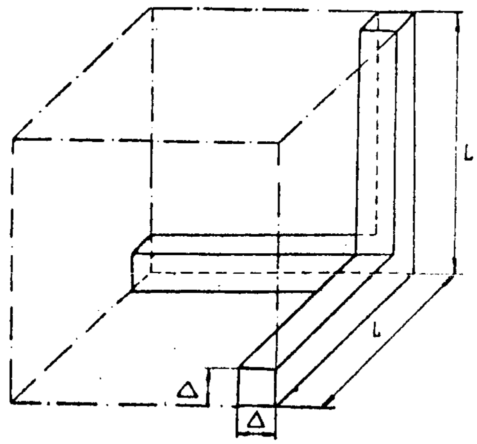
Fig.1. Schematic representation of the structure of a solid with communicating pores :

a) skeleton of beams with constant cross section;b) skeleton of beams with variable cross section; c) skeleton of the type a, but with pores shifted relative to one another.

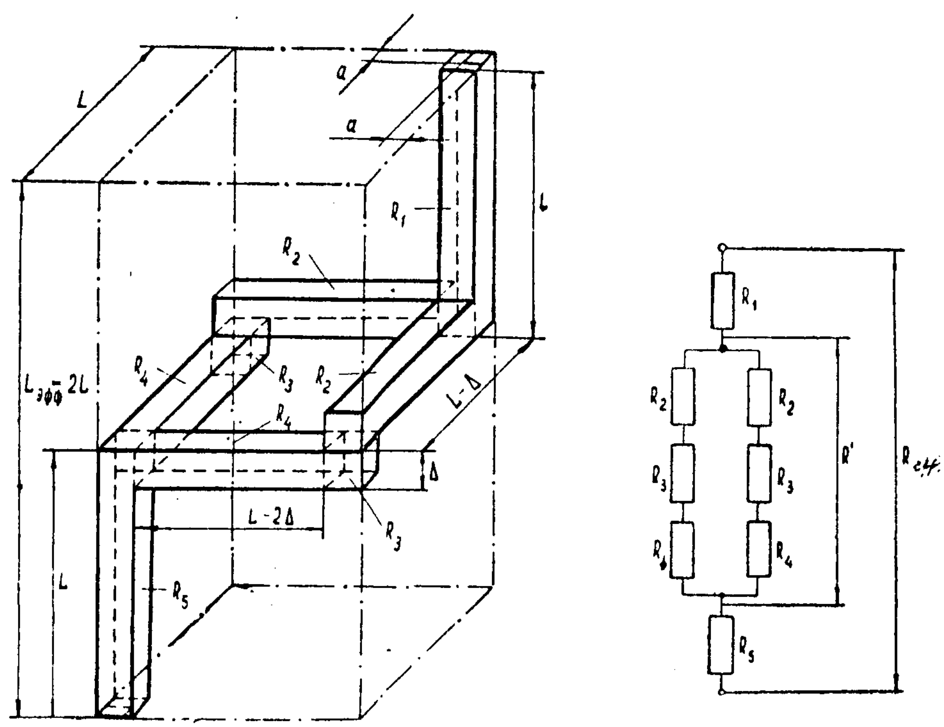
Let us conditionally break down the beams in the elementary mesh into separate parts (1 - 5) (Fig. 2b) and compute their thermal resistances, assuming that the lines of the thermal flux in separate beams are mutually parallel. In this case the thermal resistance of separate parts of the mesh may be determined according to the formula for a plane wall:

$$R_i = L_i/\lambda_1 S_i, \tag{2}$$

where L_i and S_i are respectively the length and the area of the cross section of the i-th part of the mesh, λ_1 is the heat conductivity of the skeleton.



a



b c

Fig.2. Elementary mesh for different structures

a) structure of Fig.1a,b;b) structure of Fig.1c;
c) scheme of combination of thermal resistances in an elementary mesh

From Fig. 2b and formula (2) it may be seen that

$$R_1 = L/\lambda_1\Delta^2, \quad R_2 = \frac{L-\Delta}{\lambda_1\Delta^2}, \quad R_3 = \Delta/\lambda_1\Delta^2, \quad R_4 = \frac{L-2\Delta}{\lambda_1\Delta^2}. \tag{3}$$

The scheme of the combination of thermal resistances of separate parts R_i, entering into the elementary mesh is represented in Fig.2c. The total (effective) resistance R_{eff} of an elementary mesh is not difficult to determine:

$$R_{eff} = 2R_1 + R', \quad R' = \frac{1}{\sigma'}, \quad \sigma' = 2(\sigma_2 + \sigma_3 + \sigma_4), \quad \sigma_i = \frac{1}{R_i}. \tag{4}$$

From Eqs. (3) and (4) we obtain

$$R_{eff} = \frac{3L-\Delta}{\lambda_1 S_{\text{вх}}} = \frac{3L-\Delta}{\lambda_1\Delta^2}. \tag{5}$$

If we utilize the notion of effective heat conduction factor (λ_{eff}) of an elementary mesh, its thermal resistance may be represented in the form

$$R_{eff} = \frac{L_{eff}}{\lambda_{eff}\, S_{eff}} = \frac{2}{\lambda_{eff}\, L}, \tag{6}$$

where L_{eff} and S_{eff} are the length and the area of the elementary mesh.

Equating Eqs. (5) and (6), we obtain the expression for the effective heat conductivity of the skeleton with beams of constant cross sections with shifted pores:

$$\lambda_{eff} = \lambda_1 x^2 f(x), \qquad f(x) = \frac{2}{3-x} \tag{7}$$

At the same time parameter x in formulas (1) and (7) is linked with the porosity p by the dependence [3]

$$p = 1 + 2x - 3x, \quad p = \frac{V_{por}}{V} = \frac{\rho_1}{\rho_2} \tag{8}$$

where V_{por} and V are respectively the volume of pores in the body and the total volume of the body; ρ_1 and ρ_2 are the volumetric densities of the porous body and of the skeleton.

The accounting for pore shift in a solid body with communicating pores decreases the effective heat conductivity; the values of λ_{eff} computed by formulas (1) and (7) differ by less than 30%. Consequently, such structures still are not intermediate between the granular system and a solid porous body.

The cases considered refer to structures with beams of constant cross section. In dendritic structures the area of conducting phase's cross section vary sharply, forming peculiar narrowings - a sort of neck. Assume now that the thermal flux enters the elementary mesh and exits from it only through the neck with cross section area $S_{\text{вх}} = S_{neck} = a^2$, smaller than the cross section area $S_\sigma = \Delta^2$ (Fig.2b). Let us substitute the skeleton of the elementary mesh consisting of beams with constant cross section, by a cylinder of reduced length l, whereupon the thermal resistance of such a cylinder, R_c is equal to the effective thermal resistance R_{eff} of the elementary mesh:

$$R_c = \frac{l}{\lambda_c S_c} = R_{eff} = \frac{3L - \Delta}{\lambda_1 \Delta^2} \tag{9}$$

(we shall consider that $S_c = \Delta^2$, $\lambda_c = \lambda_1$). From formula (9) we obtain the expression for the reduced length l of the cylinder:

$$l = 3L - \Delta. \tag{10}$$

In this case the thermal flux enters the extremity of the cylinder through the surface of which the area is $S_{neck} = a^2$, and emerges from the opposite end through a surface of identical area; we shall consider the remaining surfaces as adiabatic (Fig.3). The distortion of the thermal current lines leads to the variation of cylinder's thermal resistance.

If the mean surface temperatures of the neck at inlet and outlet are respectively $\bar{t}_1$ and $\bar{t}_2$, and the thermal flux capacity is P, the thermal resistance R of the cylinder is by definition

$$R = \frac{\bar{t}_1 - \bar{t}_2}{P}. \tag{11}$$

The temperature difference $\bar{t}_1 - \bar{t}_2$ is then linked with the geometrical and physical parameters of the cylinder by the dependence [7]:

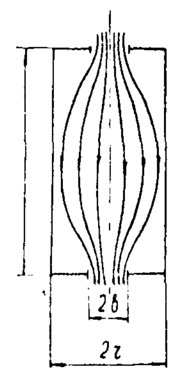


Fig.3. Thermal current lines in a part of cylindrical beam of variable cross section

$$\bar{t}_1 - \bar{t}_2 = \frac{Pl}{\lambda_1 S}\left\{1 - \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{J_1(n\pi b/l)}{n^2 J_1(n\pi r/l)}\left[J_1(n\pi r/l)\,K_1(n\pi b/l) - K_1(n\pi r/l)\,J_1(n\pi b/l)\right]\right\}, \tag{12}$$

where n = 2k + 1, k = 0, 1, 2, 3, ..., P is the total thermal flux, S is the area of the neck, l, r, b are respectively the length and the radius of the cylinder and the radius of the neck, λ_1 is the cylinder's thermal conductivity factor, J_1 is a Bessel function of first order of imaginary argument, K_1 is a MacDonald function of first order.

From expressions (11) and (12) follows

$$R = R_0 \cdot \Phi(b/r,\ r/l), \quad R_0 = l/\lambda_1 S, \tag{13}$$

where R is the thermal resistance of the cylinder with area $S = \pi b^2$,

$$\Phi(b/r,\ r/l) = 1 - 16/\pi^2 \sum_{n=1}^{\infty} [J_1(n\pi b/l)] / [n^2 J_1(n\pi r/l)] \times$$
$$\times [J_1(n\pi r/l)\, K_1(n\pi b/l) - K_1(n\pi r/l)\, J_1(n\pi b/l)]. \qquad (14)$$

The values of the function $\Phi(b/r,\ r/l)$, computed by formula (11) for a series of relations b/r and r/l, are represented in Fig.4.

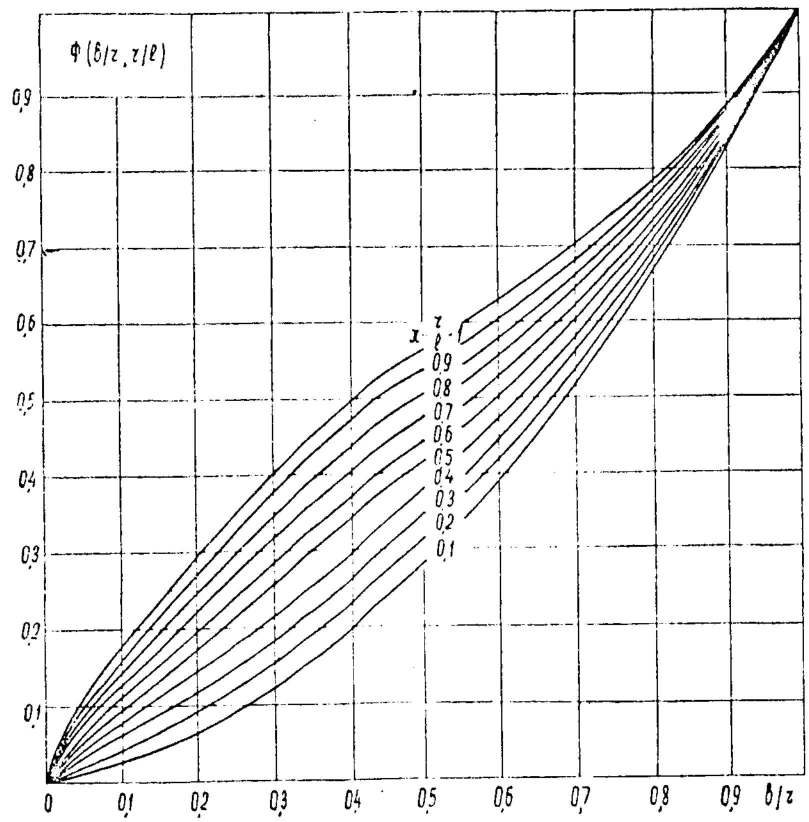


Fig.4 Graph of the function (r/l, b/r)

Let us substitute the cylinder a by a beam of rectangular cross section, at the basis of which lies a square with the side Δ, and let us consider that the thermal flux enters and departs from it through the transverse cross section of a square with side a. At the same time $\pi b^2 = a^2$, $\pi r^2 = \Delta^2$. Then the function $\Phi(b/r,\ r/l) = \Phi(\bar{a}/\Delta,\ \Delta/l)$ characterizes the variation of beam's resistance on account of the variation of the transverse cross section.

The value of the thermal resistance R_0 of the beam of constant cross section may be determined by formula (5):

$$R_0 = l/\lambda_1 S_{\text{ux}} = l/\lambda_1 a^2. \qquad (15)$$

The value of the thermal resistance of the beam with variable cross section is determined from expressions (13) and (15):

$$R=\frac{l}{\lambda_1 a^2}\Phi\left(\frac{a}{\Delta},\ \frac{\Delta}{l}\right). \qquad (16)$$

The effective resistance of an elementary mesh for a system with pore shift and beams of variable cross section may be expressed in the form (see Fig.2b)

$$R_{eff} = 2/\lambda_{eff} L. \qquad (17)$$

Equating (16) and (17) as was done in the case of pore shift, we shall obtain the expression for the effective heat conductivity of dendritic structure:

$$\lambda_{eff} = \lambda_1 x^2 y^2 \frac{f(x)}{\Phi(y,\ \Delta/l)},\quad y=a/\Delta. \qquad (18)$$

For small values of y (beginning with $y \leqslant 5 \cdot 10^{-2}$) the determination of the function $\Phi(y,\ \Delta/l)$ from the graph leads to a great error. To diminish it the dependence between $\Phi(y,\ \Delta/l)$ and $\underline{y}$ may be approximated by a linear function of the form

$$\Phi(y,\ \Delta/l) = 2.22y\ \Delta/l \quad (y \leqslant 5 \cdot 10^{-2}) \qquad (19)$$

It follows from Eqs (18) and (19) that

$$\lambda_{eff} = 0.9\ \lambda_1 xy. \qquad (20)$$

All the above analytical dependences refer to a model of dendritic structure with monolithic skeleton.

Let us consider now a dendritic structure having a porous skeleton with communicating pores of second order of smallness (structure close to the natural volcanic tuff). Let us subsdivide the total porosity of the diphase system $\underline{p}$ into the external p_1 and the internal p_2 :

$$p = p_1 + p_2 \qquad (21)$$

where

$$p = V_{por}/V, \quad p_1 = V_{por\ 1}/V, \quad p_2 = V_{por\ 2}/V,$$

V_{por} being the volume of all pores, $V_{por\ 1}$ that outside the skeleton, $V_{por\ 2}$ the volume of pores inside the skeleton. V is the total volume of an elementary mesh.

Let us introduce the notion of skeleton porosity

$$p_{sk} = V_{por\ 2}/V_{sk}, \qquad (22)$$

where V_{sk} is the total volume of the skeleton.

It may be shown that

$$p_{sk} = \frac{p-p_1}{1-p}. \tag{23}$$

Then the calculation of the effective heat conductivity is performed in two stages. In the first stage the heat conductivity of the porous skeleton is determined by formulas (7) and (8) as a function of its porosity p_{sk}. The value obtained for the heat conductivity of a hard skeleton λ_{sk} is substituted into formula (18) instead of λ_1 and the effective heat conductivity of the dendritic structure is determined as a whole. Eq. (7) may then be written in the form

$$\lambda_{sk} = \lambda_1 x_2^2 f(x_2), \tag{24}$$

and the effective heat conductivity (18) may then be expressed as follows:

$$\lambda_{eff} = \lambda_1 x_1^2 x_2^2 y^2 \frac{f(x_1)\, f(x_2)}{\Phi(y, \Delta/l)}, \tag{25}$$

where x_1 is the root of Eq, (8), composed for $p = p_1$, x_2 is the root of Eq. (8) for $p = p_{sk}$.* Then Eq. (23) takes the form

$$p_1^2 - 2p_1 + p = 0,$$

whence it follows that

$$p_1 = 1 - \sqrt{1-p}. \tag{26}$$

Taking into account the condition $p_1 = p_{sk}$, we obtain from formula (25) the minimum value of the effective heat conductivity:

$$\lambda_{eff\ min} = \lambda_1 x^4 y^2 [f(x)]^2 / \Phi(y, \Delta/l). \tag{27}$$

Let us perform by the method proposed the calculation of the effective heat conductivity of a volcanic tuff in vacuum. A direct consideration of a volcanic tuff with the aid of a stereomicroscope has shown that the tuff has a dendritic structure and a porous skeleton. The heat conductivity of a volcanic tuff is $\lambda_1 = 2.5\ w{\cdot}m^{-1}\ deg^{-1}$, the volume density of the skeleton without pores is $\rho_1 = 2.5 \cdot 10^3\ kg{\cdot}m^{-1}$ [10]. The most probable density of the tuff in "lunar conditions" would be $\rho = 400 \div 800\ kg\ m^{-1}$ [5]. The calculation of λ_{eff} is performed according to formulas (8) and (18) for various values of y. The results of the calculation of $\lambda = \lambda(\rho)$ are plotted in Fig. 5.

As an example we shall consider the calculation of the dependence of $\lambda_{eff} = \lambda(\rho)$ for the case $\rho = 600\ kg{\cdot}m^{-1}$, $y = 0.3$. By formula (10) we shall determine the general formula of tuff porosity :

$$p = 0.76.$$

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* Because of the subdivisions we obtain different values for λ_{eff} of a dendritic system, and the minimum, $\lambda_{eff\,min}$ is obtained for $p_1 = p_{sk}$.

Assuming $p_1 = p_{sk}$, we shall find the external porosity:

$$p_1 = 0.51.$$

From Eq. (8) we find the value $x = 0.49$ corresponding to porosity p_1. and by formulas (1) and (10), the value $\Delta/l = 0.195$. Then, assigning the relative magnitude of narrowings $y = 0.3$, we find by the graph of Fig.4 the value

$$\Phi(y, \Delta/l) = 0.15$$

for $y = 0.3$ and $\Delta/l = 0.195$.

By formula (7) we find $f(x) = 0.8$ and by formula (27) we shall determine the minimum value of the effective heat conductivity of the chosen structure:

$$\lambda_{eff} = 0.055\ \mathrm{w \cdot m^{-1} \cdot deg^{-1}}.$$

As recalled above, the results of astrophysical measurements of the quantity γ and the assumption relative to the density and the heat conductivitv of the lunar surface layer's matter will lead to the values of the effective heat conductivity factor $\lambda_{eff} = 0.04\ \mathrm{w \cdot m^{-1} \cdot deg^{-1}}$. Choosing other values of relative narrowing y, it is possible to obtain the various values of λ_{eff} (for example, when $y = 1$, $\lambda_{eff} = 0.09$; for $y = 0.1$, $\lambda_{eff} = 0.02$; for $y = 0.01$, $\lambda_{eff} = 0.01$; for $y = 0.001$, $\lambda_{eff} = 0.001$). In other words, it is possible to encompass all the intermediate values of the effective heat conductivity factor from a solid body with communicating pores and skeleton with beams of constant cross section to granular systems.

The solution of the inverse problem (determination of body's structure by its effective heat conductivity) requires additional data on the structure of the body: it is necessary to make assumptions about body porosity, the relative value of skeleton narrowing and so forth. Consequently, the knowledge of only one parameter λ_{eff} does not allow us to bring forth an unambiguous judgment on the structure of the body. Nevertheless, the method considered above may be eventually useful in the case of complex study of the problem.

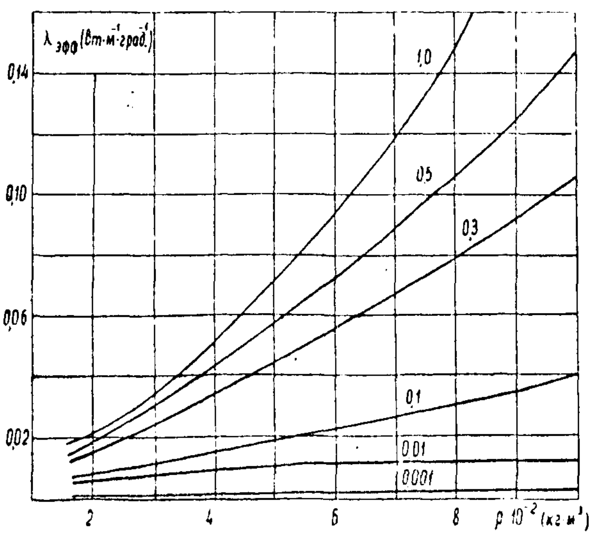

Fig.5. Graph of the dependence of the effective tuff's heat conductivity on the density and the form of the structure

*** THE END ***

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